Reliability-based design optimization of an imperfect submarine pressure hull

V. Dubourg[1,2], J.-M. Bourinet[1], B. Sudret[1,2], M. Cazuguel[3]

[1] *Clermont Université, IFMA, EA 3867, Laboratoire de Mécanique et Ingénieries, BP 10448 F-63000 Clermont-Ferrand*
[2] *Phimeca Engineering, Centre d'Affaires du Zénith, 34 rue de Sarlième, F-63800 Cournon d'Auvergne*
[3] *DCNS, Ingénierie Navires Armés, rue Choiseul, F-56311 LORIENT Cedex*

ABSTRACT: Reliability-based design optimization (RBDO) has gained much attention in the past fifteen years as a way of introducing robustness in the process of designing structures and systems in an optimal manner. Indeed classical optimization (*e.g.* minimize some cost under mechanical constraints) usually leads to solutions that lie at the boundary of the admissible domain, and that are consequently rather sensitive to uncertainty in the design parameters. In contrast, RBDO aims at designing the system in a robust way by minimizing some cost function under reliability constraints. Thus RBDO methods have to mix optimization algorithms together with reliability calculations. The classical approach known as "double-loop" consists in nesting the computation of the failure probability with respect to the current design within the optimization loop. It is not applicable to industrial models (*e.g.* finite element models) due to the associated computational burden. In contrast, methods based on the approximation of the reliability (*e.g.* FORM) may not be applicable to real-world problems. In this context, an original method has been developed that tries to circumvent the abovementioned drawbacks of the existing approaches. It is based on the adaptive construction of a *meta-model* for the expensive-to-evaluate mechanical model, and on the *subset simulation* technique for the efficient and accurate computation of the failure probability and its sensitivities with respect to the design variables. The proposed methodology is briefly described in this paper before it is applied to the reliability-based design of an imperfect submarine pressure hull.

1 INTRODUCTION

In order to ensure that their design is safe enough, mechanical engineers and other stake-holders usually cope with uncertainty using *partial safety factors*. Such factors are usually prescribed in specific standards depending on the field of application and are calibrated with respect to both experience and scatter of environmental, material and structural properties. However, they might either introduce an excessive degree of conservatism or on the contrary, they might lack exhaustivity about the servicing conditions of the system to be designed. This is the reason why Reliability-Based Design Optimization has gained much attention in the past decades as it allows one to consider explicitly uncertainty in the design optimization procedure.

RBDO aims at designing the system in a robust manner by minimizing some cost function c with respect to a set of characteristic design parameters $\boldsymbol{\theta}$ (*e.g.* means) under *reliability constraints* instead of the original *deterministic constraints*. The reliability-based optimal design $\boldsymbol{\theta}^*$ of interest is thus defined as the following minimizer:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}\in\mathcal{D}_{\boldsymbol{\theta}}} c(\boldsymbol{\theta}) \; : \; \begin{cases} f_i(\boldsymbol{\theta}) \leq 0, \; i = 1, \ldots, n_c \\ \mathbb{P}\left(g_l\left(\boldsymbol{X}(\boldsymbol{\theta})\right) \leq 0\right) \leq P_{fl}^0, \; l = 1, \ldots, n_p \end{cases} \tag{1}$$

where $\mathcal{D}_{\boldsymbol{\theta}}$ is the admissible design space, $\{f_i, i = 1, \ldots, n_c\}$ are classical deterministic constraints that either do not put the system's safety at stake or involves partial safety factors in order to cope with some unconsidered failure scenarii, $\{\mathbb{P}\left(g_l\left(\boldsymbol{X}(\boldsymbol{\theta})\right) \leq 0\right) \leq P_{fl}^0, l = 1, \ldots, n_p\}$ are the so-called *reliability constraints* involving the vector of uncertain variables $\boldsymbol{X}$ and the maximum failure probabilities that are tolerated by the code of practice $\{P_{fl}^0, l = 1, \ldots, n_p\}$. Note that, in this paper, $\boldsymbol{X}$ is distributed according to its joint probability density function $f_{\boldsymbol{X}}(\bullet, \boldsymbol{\theta})$ which is parameterized by the design parameters $\boldsymbol{\theta}$.

In essence, RBDO methods have to mix optimization algorithms together with reliability calculations. The classical approach known as "*double loop*" consists in nesting the computation of the probability of failure with respect to the current design within the optimization loop. Crude or advanced simulation methods are not applicable to industrial models (*e.g.* finite element models) due to the associated computational burden. In contrast, methods based on the approximation of the reliability (*e.g.* FORM) may not be applicable to real-world problems. The interested reader may refer to the book by Tsompanakis et al. (2008) which provides a complete review of existing approaches.

In order to try to circumvent the aforementioned drawbacks of existing approaches, an original method has been developed. It is based on both the adaptive construction of a *meta-model* for the original time-consuming mechanical model, and the use of the *subset simulation* technique for the efficient and accurate computation of the failure probability and its associated sensitivities w.r.t. the design variables. The strategy is briefly described in this paper before its application to the design of imperfect submarine pressure hulls. The interested reader may find some more detailed explanations in a forthcoming paper (Dubourg et al., 2011). The case addressed in this paper aims at finding the optimal dimensions of a single bay of a ring-stiffened cylinder representative of a submarine pressure hull under external hydrostatic pressure. It accounts for uncertainties in the material properties and in the amplitudes of its imperfections. These imperfections are supposed distributed over the two most critical buckling modes in a first simplified approach (overall and interframe buckling modes). Indeed, the work presented in this paper partly relies on closed-form and semi-analytical formulas that are commonly used for preliminary design in the submarine industry (*e.g.* the BS5500). The results provided in this paper should provide a first sound basis for a future application involving a higher fidelity nonlinear shell finite element model.

2 THE KRIGING SURROGATE

In computer experiments, *surrogate modelling* addresses the problem of replacing a time-consuming mathematical model $\mathcal{M}$ called a *simulator* by an *emulator* that is much faster to evaluate. The emulator has the same input space $\mathcal{D}_{\boldsymbol{x}} \subseteq \mathbb{R}^n$ and output space $\mathcal{D}_y \subseteq \mathbb{R}$ as the simulator. For the sake of simplicity only scalar-output simulators are considered here. It is important to note that both $\boldsymbol{x}$ and y are deterministic in this context. The emulator is usually built from a set of observations named a *design of experiments* (DOE) $\{y_i = \mathcal{M}(\boldsymbol{x}_i),\ i = 1, \dots, m\}$.

Kriging (Santner et al., 2003) is one particular emulator that is able to give a probabilistic response $\widehat{Y}(\boldsymbol{x})$ whose variance (scatter) depends on the quantity of available knowledge. In other words, the uncertainty in the prediction is purely epistemic and thus reducible as shown in the next sections.

In essence, kriging assumes that the output $y = \mathcal{M}(\boldsymbol{x})$ is a sample path of an underlying Gaussian stochastic process $Y(\boldsymbol{x})$ that would read as follows:

$$Y(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x})^{\mathsf{T}} \boldsymbol{\beta} + Z(\boldsymbol{x}) \tag{2}$$

where:

- $\boldsymbol{f}(\boldsymbol{x})^{\mathsf{T}} \boldsymbol{\beta}$ denotes the mean of the GP which corresponds to a classical linear regression model on a given functional basis $\{f_i,\ i = 1, \dots, p\} \in \mathcal{L}_2(\mathcal{D}_{\boldsymbol{x}}, \mathbb{R})$;
- $Z(\boldsymbol{x})$ denotes the stochastic part of the GP which is modelled as a zero mean, constant variance σ_Y^2, stationary Gaussian process with a given symmetric positive definite autocorrelation model. It is fully defined by its autocovariance function which reads ($(\boldsymbol{x}, \boldsymbol{x}') \in \mathcal{D}_{\boldsymbol{x}} \times \mathcal{D}_{\boldsymbol{x}}$):

$$C_{YY}(\boldsymbol{x}, \boldsymbol{x}') = \sigma_Y^2 R\left(\left|\boldsymbol{x} - \boldsymbol{x}'\right|, \boldsymbol{\ell}\right) \tag{3}$$

  where $\boldsymbol{\ell}$ is a vector of parameters defining R.

The most widely used class of autocorrelation functions is the anisotropic squared exponential model:

$$R\left(\left|\boldsymbol{x} - \boldsymbol{x}'\right|, \boldsymbol{\ell}\right) = \exp\left(\sum_{k=1}^{n} -\left|\frac{x_k - x_k'}{\ell_k}\right|^2\right) \tag{4}$$

The best linear unbiased estimation of the GP Y at point $\boldsymbol{x}$ is shown (Santner et al., 2003) to be the Gaussian random variate $\widehat{Y}(\boldsymbol{x}) \sim \mathcal{N}\left(\widehat{Y}(\boldsymbol{x}), \sigma_{\widehat{Y}}(\boldsymbol{x})\right)$ with moments:

$$\widehat{Y}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x})^{\mathsf{T}} \widehat{\boldsymbol{\beta}} + \boldsymbol{r}(\boldsymbol{x})^{\mathsf{T}} \mathbf{R}^{-1} \left(\boldsymbol{Y} - \mathbf{F}\widehat{\boldsymbol{\beta}}\right) \tag{5}$$

$$\sigma_{\widehat{Y}}^2(\boldsymbol{x}) = \sigma_Y^2 \left(1 - \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}) \\ \boldsymbol{r}(\boldsymbol{x}) \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \mathbf{0} & \mathbf{F}^{\mathsf{T}} \\ \mathbf{F} & \mathbf{R} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}) \\ \boldsymbol{r}(\boldsymbol{x}) \end{bmatrix}\right) \tag{6}$$

where we have introduced $\boldsymbol{r}$, $\mathbf{R}$ and $\mathbf{F}$ such that:

$$r_i = R\left(\left|\boldsymbol{x} - \boldsymbol{x}_i\right|, \boldsymbol{\ell}\right),\ i = 1, \dots, m \tag{7}$$

$$R_{ij} = R\left(\left|\boldsymbol{x}_i - \boldsymbol{x}_j\right|, \boldsymbol{\ell}\right),\ i = 1, \dots, m,\ j = 1, \dots, m \tag{8}$$

$$F_{ij} = f_i\left(\boldsymbol{x}_j\right),\ i = 1, \dots, p,\ j = 1, \dots, m \tag{9}$$

At this stage one can easily prove that $\widehat{Y}(\boldsymbol{x}_i) = \mathcal{M}(\boldsymbol{x}_i)$ and $\sigma_{\widehat{Y}}(\boldsymbol{x}_i) = 0$ for $i = 1, \dots, m$, thus meaning the kriging surrogate is an *exact interpolator*.

Given a choice for the regression and correlation models, the optimal set of parameters $\boldsymbol{\beta}^*$, $\boldsymbol{\ell}^*$ and $\sigma_{\widehat{Y}}^{2*}$ can then be inferred using the *maximum likelihood principle* applied to the unique sparse observation of the GP sample path grouped in the vector

$\boldsymbol{y} = \langle \mathcal{M}(\boldsymbol{x}_i),\ i = 1, \ldots, m \rangle$. This optimization problem can be solved analytically for both $\boldsymbol{\beta}^*$ and $\sigma^{2*}_{\widehat{Y}}$ assuming $\boldsymbol{\ell}^*$ is known. Thus, the problem is solved in two steps: the maximum likelihood estimation of $\boldsymbol{\ell}^*$ is first solved by a global optimization algorithm which in turns allows one to evaluate the optimal $\boldsymbol{\beta}^*$ and $\sigma^{2*}_{\widehat{Y}}$.

3 THE PROPOSED ADAPTIVE REFINEMENT TECHNIQUE

Various refinement techniques have been proposed in the kriging-related literature, especially in structural reliability (Bichon et al., 2008; Lee and Jung, 2008). They are all based on the genuine probabilistic nature of the kriging prediction. The refinement technique consists in finding the set of points that should be added to the DOE in order to improve the prediction and reduce its associated epistemic uncertainty in a specific region of interest.

In reliability and reliability-based design optimization, the region of interest in the space of random variables $\boldsymbol{X}$ is the limit-state surface $g = 0$ – we drop the subscript l in this section for the sake of clarity. Provided an initial kriging prediction of the performance function g, the probable vicinity of the limit-state surface $g = 0$ can be defined in terms of the following *margin of uncertainty*:

$$\mathbb{M} = \left\{ \boldsymbol{x} \,:\, -k\,\sigma_{\widehat{G}}(\boldsymbol{x}) \leq \widehat{G}(\boldsymbol{x}) \leq +k\,\sigma_{\widehat{G}}(\boldsymbol{x}) \right\} \tag{10}$$

where k is related to the confidence level on the prediction of the limit-state. Any point that belongs to this margin corresponds to an uncertain sign of the g-function. Thus the scatter of this margin should be reduced in order to make the reliability estimation accurate. This is achieved by adding new points of the experimental design in this margin.

The probability that a point $\boldsymbol{x} \in \mathcal{D}_{\boldsymbol{x}}$ falls in the margin $\mathbb{M}$ (with respect to the epistemic uncertainty in the prediction) can be expressed in closed-form as follows:

$$\begin{aligned} \mathbb{P}(\boldsymbol{x} \in \mathbb{M}) = \Phi\left(\frac{k\,\sigma_{\widehat{G}}(\boldsymbol{x}) - \widehat{G}(\boldsymbol{x})}{\sigma_{\widehat{G}}(\boldsymbol{x})} \right) \\ - \Phi\left(\frac{-k\,\sigma_{\widehat{G}}(\boldsymbol{x}) - \widehat{G}(\boldsymbol{x})}{\sigma_{\widehat{G}}(\boldsymbol{x})} \right) \end{aligned} \tag{11}$$

Two strategies may be used in order to add new points:

- maximise the probability in Eq. (11);
- define a sampling density that concentrates in the margin and sample according to this density, as proposed in Dubourg et al. (2010).

To do so, a sampling density is defined as the product of the probability to fall in the margin of uncertainty (Eq.(11)) and some weighting PDF w:

$$\mathcal{C}(\boldsymbol{x}) \propto \mathbb{P}(\boldsymbol{x} \in \mathbb{M})\, w(\boldsymbol{x}) \tag{12}$$

The latter equation may be considered as a PDF up to an unknown but finite normalizing constant. The weighting density w is selected here as a uniform PDF on a sufficiently large confidence region of the original PDF. Given that pseudo-PDF, one can generate N (say $N = 10^4$) samples by means of any well suited *Markov chain Monte Carlo* (MCMC) simulation technique (Robert and Casella, 2004), such as the *slice sampling* technique (Neal, 2003).

The N generated candidates are expected to be highly concentratred at the modes of $\mathcal{C}$ – and thus in $\mathbb{M}$. This population of candidates is then reduced to a smaller one that uniformally spans the margin $\mathbb{M}$ by means of the K-means clustering technique (Elkan, 2003).

Figure 1 illustrates the proposed refinement strategy applied to a nonlinear limit-state surface. The upper subfigures show the contours of the refinement pseudo-PDF $\mathcal{C}$ at each refinement step together with the candidate population generated by slice sampling and its K cluster centers – $K = 10$ being given. In the lower subfigures one can see the real limit-state surface represented as the dashed black curve, its kriging prediction represented as the black line and its associated margin of uncertainty $\mathbb{M}$ which is bounded below by the red line and above by the blue line. Another interpretation of these figures is that any point within the blue bounded shape is positive with a 95% confidence and any point inside the red bounded shape is negative with the same confidence level.

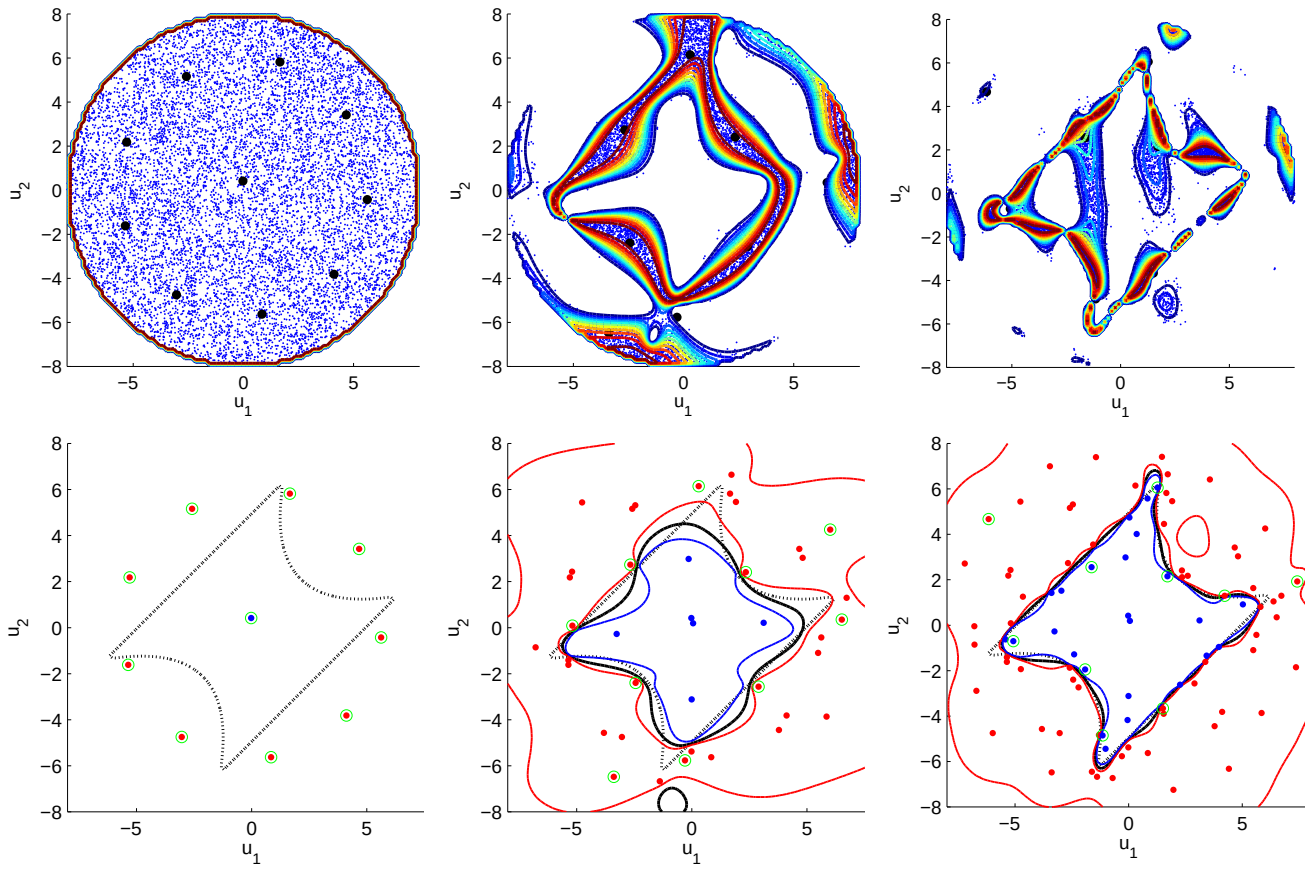
In order to stop the iterative refinement algorithm, a measure of the scatter of the margin is necessary. Introducing the three approximate failure subsets defined as $\widehat{F}^i = \{\boldsymbol{x} \,:\, \widehat{G} + i\,k\,\sigma_{\widehat{G}}(\boldsymbol{x}) \leq 0\}$, $i = -1,\, 0,\, +1$, the following statement holds:

$$\begin{aligned} &\widehat{F}^{+1} \subseteq \widehat{F}^{\,0} \subseteq \widehat{F}^{-1} \\ &\quad \Rightarrow \mathbb{P}\left(\boldsymbol{X} \in \widehat{F}^{+1}\right) \leq \mathbb{P}\left(\boldsymbol{X} \in \widehat{F}^{\,0}\right) \leq \mathbb{P}\left(\boldsymbol{X} \in \widehat{F}^{-1}\right) \end{aligned} \tag{13}$$

The spread of the interval $[\mathbb{P}(\boldsymbol{X} \in \widehat{F}^{+1}),\ \mathbb{P}(\boldsymbol{X} \in \widehat{F}^{-1})]$ is then used in this work as a stopping criteria for the proposed refinement procedure. Note that due to the inconvenient order of magnitude of these probabilities, a $\log_{10}$-scaling is used in order to assess the spread of this interval. Note also that it is neither proved or even stated that the real failure probability belongs to this interval – due to the empirical assumptions relative to the meta-modeling procedure. However the overall strategy has proved to be accurate for some applications (Dubourg et al., 2010, 2011).

4 THE PROPOSED ADAPTIVE SURROGATE-BASED RBDO ALGORITHM

The proposed strategy to solve the RBDO problem in Eq. (1) consists in nesting the previously introduced kriging surrogate together with the proposed refinement strategy within a classical (but efficient) nested RBDO algorithm.



(a) Initialization: 10-point DOE (b) Step #3: 40-point DOE (c) Step #9: 100-point DOE

Figure 1: An illustration of the proposed limit-state surface refinement technique on an example from Waarts (2000).

4.1 *The augmented reliability space*

The kriging surrogates are built in the so-called *augmented reliability space* such as defined in Taflanidis and Beck (2009). In this space, the augmented random vector $\boldsymbol{X}(\boldsymbol{\theta})$ has a PDF h which accounts for the uncertainty in the design choices and the aleatory uncertainty in the random vector $\boldsymbol{X}$. Under such considerations, h reads as follows:

$$h(\boldsymbol{x}) = \int_{\mathcal{D}_{\boldsymbol{\theta}}} f_{\boldsymbol{X}}(\boldsymbol{x}\,|\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta} \tag{14}$$

where $f_{\boldsymbol{X}}$ is the PDF of $\boldsymbol{X}(\boldsymbol{\theta})$ and π is the PDF of $\boldsymbol{\theta}$ that can be assumed uniform on $\mathcal{D}_{\boldsymbol{\theta}}$. The augmented reliability space is illustrated in Figure 2 in the case of a Gaussian variable with a uniformally distributed mean.

The DOE should uniformally span a sufficiently large *confidence region* $\mathcal{D}_{\boldsymbol{X}(\boldsymbol{\theta})}$ of this augmented PDF in order to make the surrogate limit-state surfaces accurate anywhere it may be evaluated. More precisely, it is expected to be accurate for extreme design choices $\boldsymbol{\theta}$ (*i.e.* located onto the boundaries of the optimization space $\mathcal{D}_{\boldsymbol{\theta}}$) and extreme values of the marginal random vector $\boldsymbol{X}$ (to be able to compute reliability indices as large as *e.g.* $\beta_0 = 8$). A confidence region is essentially the multivariate extension of the univariate concept of *confidence interval*. Under the previous general assumptions, giving a mathematical

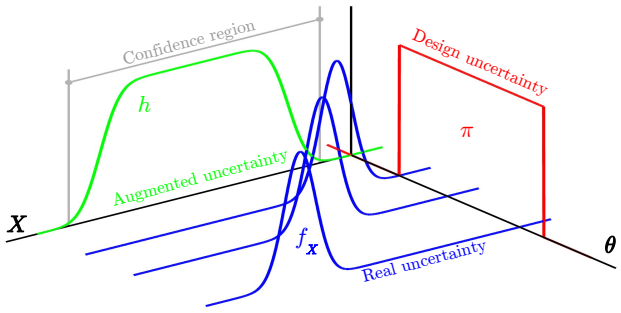


Figure 2: Illustration of the augmented reliability space.

form to the contour of this region is not as straightforward as for the standard normal space (β-radius hypersphere). However, one may easily build an hyperrectangular approximation by looking at the margins of this augmented PDF h as described in Dubourg et al. (2010).

4.2 *The adaptive surrogate-based nested RBDO algorithm*

The kriging surrogate together with its adaptive refinement procedure is finally plugged into a double-loop RBDO algorithm. The outer optimization loop is performed by means of the Polak-He optimization algorithm (Polak, 1997). Given an initial design, the algorithm proceeds iteratively on i in two steps:

(i) the direction $\boldsymbol{\delta}^{(i)}$ is determined solving a quasi-SQP sub-optimization problem and *(ii)* the step size $b^{k^{(i)}}$ is approximated by the Goldstein-Armijo approximate line-search rule. The base b in the approximate line-search is set to 0.6 and the algorithm usually steps forward with $0 \leq k \leq 10$ – provided the design variables are normalized with respect to their initial value. Note that in order to minimize the overall number of nested surrogate-based reliability analyses, the step size k is saved from one RBDO iteration to the other. The reliability and reliability sensitivity analyses are performed with the subset simulation technique (Au and Beck, 2001) onto the kriging surrogate. The subset simulation technique for reliability sensitivity analysis is detailed in Song et al. (2009). Subset simulation is used instead of crude Monte-Carlo simulation for the sake of efficiency. Indeed, even if the kriging surrogates are faster to evaluate than the original limit-state functions, they still involve a non-negligible computational effort. Namely, for a 10-variable kriging surrogate built from a DOE containing a few hundreds points, 10^5 runs of the surrogate requires an average time of 20 seconds for the mean prediction $\widehat{G}$ and 20 more seconds for the prediction variance $\sigma^2_{\widehat{G}}$. These times were measured on an Intel(R) Core(TM)2 Duo CPU (T9600 @ 2.80GHz) running with 4GB memory under Ubuntu 10.04 LTS 64bits using the built-in vectorization capacity of Matlab R2008a. 40 seconds for 10^5 complete probabilistic predictions is a lot faster than 10^5 Finite Element runs, though it is still quite time-consuming when involved in the proposed algorithm. Indeed, the DOE enrichment procedure together with the surrogate-based reliability analyses (on the three subsets to check the surrogates accuracy) are nested within the constrained optimization loop. The overall methodology was implemented within the FERUM toolbox (Bourinet et al., 2009). It makes use of the Matlab's toolboxes functions `quadprog` (for the SQP sub-optimization problem) and `slicesample` (to generate samples from the refinement pseudo-PDF $\mathcal{C}$). We give a simplified flowchart of the algorithm in Figure 3.

5 APPLICATION TO THE OPTIMAL DESIGN OF AN IMPERFECT SUBMARINE PRESSURE HULL

In this section, the proposed strategy is applied to the optimal design of an imperfect submarine pressure hull whose mechanical and stochastic models are inspired from Dubourg et al. (2008).

5.1 *Mechanical model*

The hull is modeled as a single bay of a ring-stiffened shell whose axisymmetric section is illustrated in Figure 4. This hull is supposed of infinite length and it is subject to an external hydrostatic pressure p. Indus-

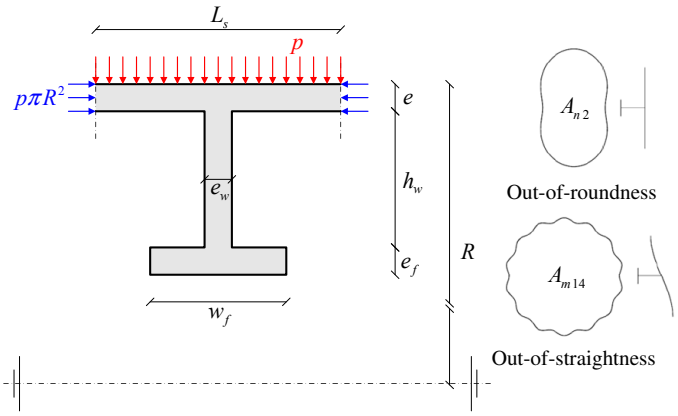


Figure 4: Single bay reference structure – Types and amplitudes of imperfections.

trial structures feature some unavoidable initial shape imperfections that are due to the manufacturing process (welding, cold bending, *etc.*), to heavy loads connected to the hull, *etc.* Even if these imperfections are of moderate amplitudes (thanks to the stringent allowance for fabrication), they may trigger buckling or premature plastic collapse at pressure far below than those corresponding to the ideal elastic buckling assumption. Predicting these collapse pressures is not straightforward and is either based on semi-analytical formulas (preliminary design) or on high fidelity non-linear finite element models (final design).

The present application is based on the semi-analytical formulas presented in Dubourg et al. (2008). The code of practice usually demands that the stake-holders validate their design with respect to three typical buckling patterns known as the overall buckling mode, the interframe buckling mode and frame tripping. The two first buckling modes are assumed to be triggered by two typical shapes of imperfection, respectively: *out-of-roundness* and *out-of-straightness*. Their amplitudes are denoted by A_n and A_m respectively. For the specific design of interest (mean values of Table 1) n and m values are respectively set to 2 and 14. These imperfections are illustrated in Figure 4 and the associated collapse pressures respectively $p_{n\,\mathrm{pl}}$ and $p_{m\,\mathrm{pl}}$ are explicited in Dubourg et al. (2008). For the last local instability though, there is no analytical formula for the collapse pressure (at least to the authors' knowledge). However, the BS5500 (section 3.6.2.2.2) provides instead two arbitrary bounds (using built-in partial safety factors) on the stiffeners' proportions (Dubourg et al., 2008).

5.2 *Stochastic model and failure scenarii*

The stochastic model is given in Table 1. All stochastic variables are assumed independent. The standard deviation of the design variables are kept constant along the optimization. For both amplitudes distributions, a lognormal distribution with a 50% coefficient of variation is assumed. The mean values are determined so that the 99.5% quantiles coincides with the

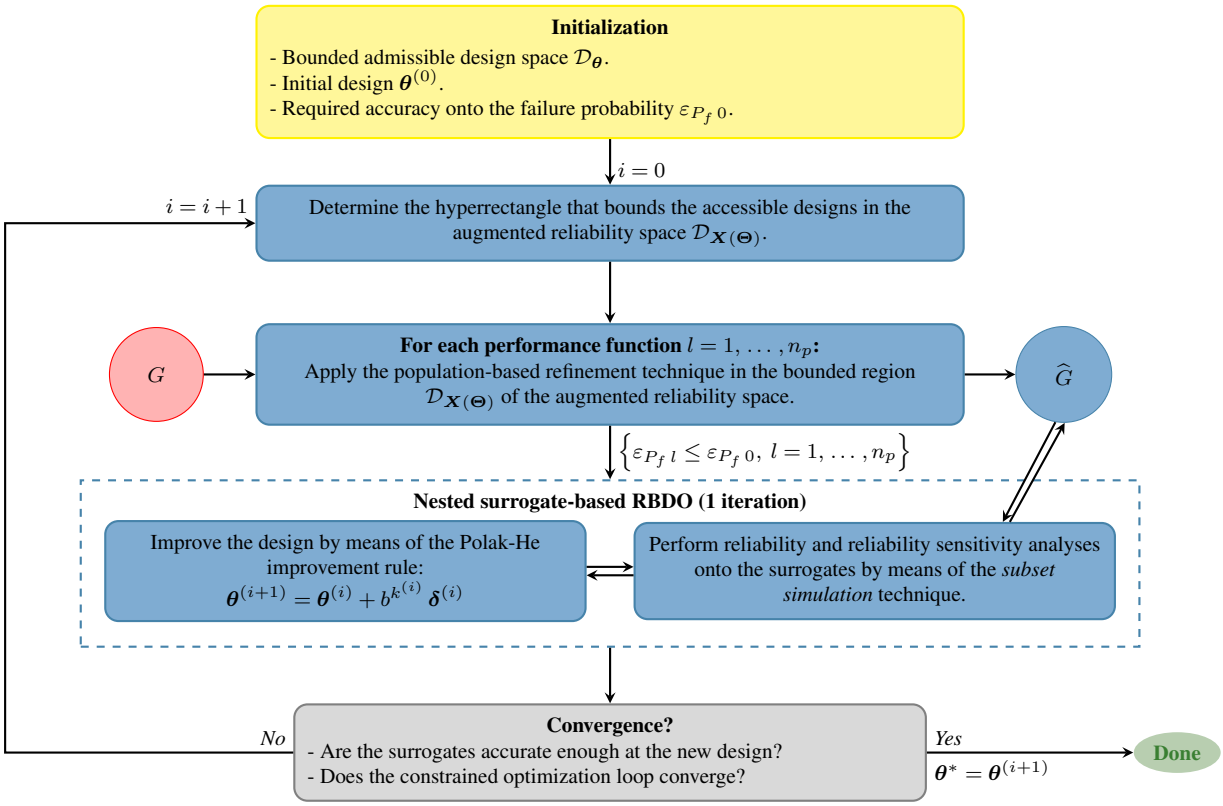


Figure 3: The proposed adaptive nested surrogate-based RBDO approach.

Variable	Distribution	Mean	St. dev.	Units
E	Lognormal	200000	10000	MPa
ν	Deterministic	0.30	–	–
σ_y	Lognormal	390	19.5	MPa
L_s	Deterministic	600	–	mm
R	Deterministic	2488	–	mm
e	Lognormal	24	0.72	mm
h_w	Lognormal	156	4.68	mm
e_w	Lognormal	10	0.30	mm
w_f	Lognormal	120	3.60	mm
e_f	Lognormal	24	0.72	mm
$A_{n\,2}$	Lognormal	$5R/3000$	$0.5 \times 5\,R/3000$	mm
$A_{m\,14}$	Lognormal	$L_s/300$	$0.5 \times L_s/300$	mm
p_0	Deterministic	2	–	MPa

Table 1: Stochastic model for the single bay reference structure.

maximum allowable values $A_{n\,2} = 5\,R/1000$ (BS5500) and $A_{m\,14} = L_s/100$ (choice of the authors). Due to the lognormal assumption this is achieved for a mean value equal to about the third of the targeted 99.5% quantile.

Two failure scenarii are considered in this paper. They are defined with respect to an arbitrary reference diving pressure $p_0 = 2$ MPa. The first failure scenario considers overall buckling and is thus described by the following limit-state function:

$$g_1(\boldsymbol{X}(\boldsymbol{\theta})) = \log \frac{p_{n\,\mathrm{pl}}}{p_0} \tag{15}$$

and the second failure scenario considers interframe buckling so that its limit-state function reads:

$$g_2(\boldsymbol{X}(\boldsymbol{\theta})) = \log \frac{p_{m\,\mathrm{pl}}}{p_0} \tag{16}$$

The logarithmic scaling of the two previous limit-states is chosen so as to simplify the meta-modeling task. Finally, the structure is considered to fail when either one or both of the two previous scenarii occurs (serial system combination), so that the limit-state function reads:

$$g(\boldsymbol{X}(\boldsymbol{\theta})) = \min\{g_1(\boldsymbol{X}(\boldsymbol{\theta})),\, g_2(\boldsymbol{X}(\boldsymbol{\theta}))\} \tag{17}$$

Note that in the proposed strategy a surrogate is built for each limit-state, and the final surrogate limit-state is evaluated as the minimum of these two.

5.3 *RBDO problem formulation*

The reliability-based optimization is carried out with respect to the mean dimensions of the stiffener: μ_e, μ_{h_w}, μ_{e_w}, μ_{e_f}, μ_{w_f}. The interframe distance L_s and the mean radius R are supposed to be imposed by other constraints (nuclear reactor size, *etc.*). The optimal mean design should minimize the expected "*hull weight*" / "*water displacement*" ratio which is approximated as follows (using the mean values of all the random variates):

$$\begin{aligned}\mathbb{E}\left[c(\boldsymbol{X}(\boldsymbol{\theta}))\right] &\approx c(\mathbb{E}\left[\boldsymbol{X}(\boldsymbol{\theta})\right]) \\ &\approx \frac{\rho_{\text{steel}}\,\mathcal{V}_{\text{steel}}(\mathbb{E}\left[\boldsymbol{X}(\boldsymbol{\theta})\right])}{\rho_{\text{sea water}}\,\mathcal{V}_{\text{sea water}}(\mathbb{E}\left[\boldsymbol{X}(\boldsymbol{\theta})\right])}\end{aligned} \tag{18}$$

where $\mathcal{V}_{\text{steel}}$ and $\mathcal{V}_{\text{sea water}}$ stand for the hull volume and the volume of displaced water, and $\rho_{\text{steel}} = 7850$ kg/m^3 and $\rho_{\text{sea water}} = 1026$ kg/m^3 are the densities of steel and sea water respectively. Finally, in addition to the reliability constraint $\beta(\boldsymbol{\theta}) > \beta_0$, we consider the two de-

Variable	Initial	DDO	**RBDO** (Present application)			**RBDO** (Dubourg et al., 2008, Table 6)			Units
			$\beta_0 = 3.00$	$\beta_0 = 4.50$	$\beta_0 = 6.00$	$\beta_0 = 3.00$	$\beta_0 = 4.50$	$\beta_0 = 6.00$	
μ_e	24.00	16.90	21.46	26.10	32.01	*20.67	*24.47	*29.82	mm
μ_{h_w}	156.00	160.27	171.25	178.35	189.22	*161.99	*197.71	*231.82	mm
μ_{e_w}	10.00	7.16	10.15	10.46	19.66	*9.40	*7.96	*9.67	mm
μ_{w_f}	120.00	81.89	89.46	95.14	103.36	*110.58	*99.08	*116.98	mm
μ_{e_f}	24.00	16.76	21.28	26.04	31.85	*22.18	*24.14	*27.01	mm
c	18.86	12.75	16.57	20.01	26.07	16.41	18.64	23.14	%
β***	4.04	-0.44	3.04	4.48	6.01	2.98	4.47	5.96	–
C.o.V.***	3.34	0.02	2.52	3.82	5.40	2.50	3.86	5.30	%
g_1-calls	–	183	200	300	350	**3257	**9494	**6580	–
g_2-calls	–	183	200	300	350				–

* The design variables were considered as deterministic. ** The authors provide only the total number of calls.
*** Reliability indices checked on the real limit-state functions using *subset simulation* with $N = 10^5$ runs/step.

Table 2: Optimal designs for the single bay reference structure.

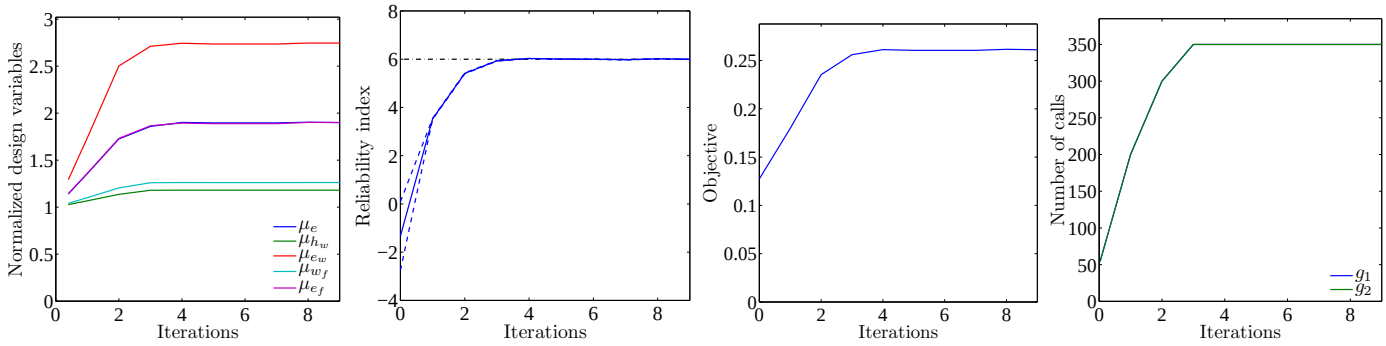


Figure 5: Convergence of the proposed algorithm for the minimum reliability constraint $\beta_0 = 6.00$.

terministic safety constraints from the BS5500 given in Dubourg et al. (2008).

5.4 *Results*

First, the starting design for the RBDO problem is determined by solving the equivalent *deterministic design optimization* (DDO) problem. To do so, all the random variates are replaced by their mean value and the probabilistic limit-states are used as deterministic constraints. The deterministic optimal design is $e = 16.90$ mm, $h_w = 160.27$ mm, $e_w = 179.86$ mm, $w_f = 81.89$ and $e_f = 16.76$. This design lies on the boundaries of the two limit-states $g_1 = 0$ and $g_2 = 0$ and is consequently rather sensitive to uncertainty as it is associated with a generalized reliability index of $\beta \approx -0.47$ (*i.e.* a failure probability greater than 50%).

Then, the proposed strategy is applied to solve the reliability-based design optimization starting from the deterministic optimal design and for three different minimum reliability constraints: $\beta_0 = \{3.00, 4.50, 6.00\}$. The algorithm uses a 50-point initial DOE and it is sequentially enriched with 50 new points if required – *i.e.* if the $\log_{10}$-scaling of the spread between $\mathbb{P}(\boldsymbol{X} \in \widehat{F}^{+1})$ and $\mathbb{P}(\boldsymbol{X} \in \widehat{F}^{-1})$ is greater than $\varepsilon_{P_f 0} = 5 \times 10^{-2}$. The results are given in Table 2 together with the initial design (which corresponds to the current practice), the deterministic optimal design and the FORM-based optimal designs from Dubourg et al. (2008).

For each design, we provide the associated objective function value, the generalized reliability index estimated by *subset simulation* (using the real limit-state function) with its *coefficient of variation* (C.o.V.), and the overall number of calls to the limit-state functions. None of the local instability safety constraints prescribed in the BS5500 are saturated at these optimized designs. Note that even if the number of calls to the limit-state functions increases with the minimum reliability target, convergence is achieved using a few hundreds evaluations of each only compared to a few thousands for the FORM-based double-loop approach proposed in Dubourg et al. (2008). The reliability-based optimal design provided in this paper are significantly less optimal than the one given in Dubourg et al. (2008). This may be explained as follows:

- the design variables were assumed deterministic in Dubourg et al. (2008) whereas they are considered as random with a 3% C.o.V. (w.r.t. the initial mean design of Table 1) in the present application;
- the FORM assumptions do not seem to be conservative in this case since the reliability indices estimated by *subset simulation* are lower than the minimum reliability constraints.

The convergence of the proposed algorithm is depicted in Figure 5 for the minimum reliability constraint $\beta_0 = 6.00$. It can be seen from the extreme-right subfigure that the number of limit-state function calls increases as the optimization algorithm moves to

higher reliability levels. It makes sense as the lower the failure probability is, the narrower the *margin of uncertainty* has to be. The extreme-left subfigure shows the convergence of the design variables normalized with respect to their starting deterministic optimal values. It means that the product of the optimal characteristic mean design (from DDO) by these optimal normalized values would ensure a β_0-reliability optimal design. They could thus be interpreted as *optimal partial safety factors*, though they were calibrated with respect to a single reference structure ($R = 2880$ mm, $L_s = 600$ mm) as code calibration usually requires to consider several reference structures.

6 CONCLUSIONS

The proposed surrogate-based approach allows to save a significant number of simulation runs without using the first-order reliability theory assumptions. It is important to note that its efficiency mainly relies upon the fact that the kriging surrogates are built in the so-called *augmented reliability space* and that they can thus be used from one nested reliability analysis to the other. The results provided in this paper provides some reference results for a future application to the design of imperfect submarine pressure hulls that will involve a higher fidelity mechanical model, namely a *nonlinear shell finite element model*. Indeed, it is of the authors' belief that reliability-based designs have to be based on the best up-to-date numerical methods, for meaningful and trustworthy results, and without depending too much on the degree of conservatism inherent to closed-form or approximate solutions such as the one used in the present paper.

ACKNOWLEDGEMENTS

The first author was funded by a CIFRE grant from Phimeca Engineering S.A. subsidized by the ANRT (convention number 706/2008). The financial support from DCNS is also gratefully acknowledged.

REFERENCES

BS5500. Unfired fusion-welded pressure vessels. British Standard Institutions.

Au, S. & J. Beck (2001). Estimation of small failure probabilities in high dimensions by subset simulation. *Prob. Eng. Mech. 16*(4), 263–277.

Bichon, B., M. Eldred, L. Swiler, S. Mahadevan, & J. McFarland (2008). Efficient global reliability analysis for nonlinear implicit performance functions. *AIAA Journal 46*(10), 2459–2468.

Bourinet, J.-M., C. Mattrand, & V. Dubourg (2009). A review of recent features and improvements added to FERUM software. In *Proc. ICOSSAR'09, Int Conf. on Structural Safety And Reliability, Osaka, Japan*.

Dubourg, V., J.-M. Bourinet, & B. Sudret (2010). A hierarchical surrogate-based strategy for reliability-based design optimization. In D. Straub, L. Esteva, and M. Faber (Eds.), *Proc. 15th IFIP WG7.5 Conference on Reliability and Optimization of Structural Systems, Munich, Germany*, pp. 53–60. Taylor & Francis.

Dubourg, V., C. Noirfalise, & J.-M. Bourinet (2008). Reliability-Based Design Optimization: An application to the buckling of imperfect shells. In *4th ASRANet colloquium*, Athens, Greece.

Dubourg, V., B. Sudret, & J.-M. Bourinet (2011). Reliability-based design optimization using kriging and subset simulation. Submitted to Struct. Multidisc. Optim.

Elkan, C. (2003). Using the triangle inequality to accelerate K-means. In *Proceedings of the Twentieth International Conference on Machine Learning (ICML-2003)*, Washington D.C.

Lee, T. & J. Jung (2008). A sampling technique enhancing accuracy and efficiency of metamodel-based RBDO: Constraint boundary sampling. *Computers & Structures 86*(13-14), 1463–1476.

Neal, R. (2003). Slice sampling. *Annals Stat. 31*, 705–767.

Polak, E. (1997). *Optimization algorithms and consistent approximations*. Springer.

Robert, C. & G. Casella (2004). *Monte Carlo statistical methods (2^{nd} Ed.)*. Springer Series in Statistics. Springer Verlag.

Santner, T., B. Williams, & W. Notz (2003). *The design and analysis of computer experiments*. Springer series in Statistics. Springer.

Song, S., Z. Lu, & H. Qiao (2009). Subset simulation for structural reliability sensitivity analysis. *Reliab. Eng. Sys. Safety 94*(2), 658–665.

Taflanidis, A. & J. Beck (2009). Stochastic subset optimization for reliability optimization and sensitivity analysis in system design. *Computers & Structures 87*(5-6), 318–331.

Tsompanakis, Y., N. Lagaros, & M. Papadrakis (Eds.) (2008). *Structural design optimization considering uncertainties*. Taylor & Francis.

Waarts, P.-H. (2000). *Structural reliability using finite element methods: an appraisal of DARS: Directional Adaptive Response Surface Sampling*. Ph. D. thesis, Technical University of Delft, The Netherlands.